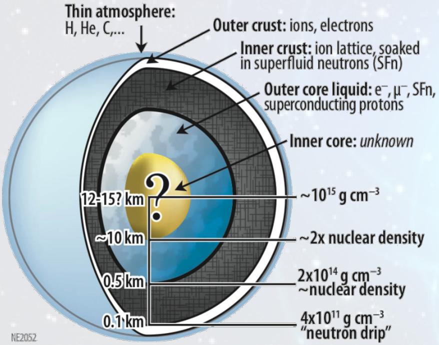
Neutron stars (NS) and nuclear superfluidity

- **Predicted** in **1933** (Baade and Zwicky) and **discovered** in **1967** (Bell and Hewish).
- Mass = 1-2 $M_\odot$ and Radius $\simeq$ 10 km $\longrightarrow$ **Density exceeding** that of **atomic nuclei** !

Neutron stars (NS) are **one of the most compact objects** in the Universe which contains **nuclear superfluids** whose existence was **predicted** in 1959 (Migdal), **before the first discovery of NS**.



- **Outer core** made of electrons and muons (at normal phase) with **superfluid** neutrons and **superconducting (~ charged superfluid)** protons.
- Nuclear superfluidity has fround **strong evidence** : Pulsar **glitches** and **NS cooling**.
- Superfluid = not one but **two velocity fields : "normal" fluid + superfluid** $\longrightarrow$ Superfluid hydrodynamics is non-classical hydrodynamics !

Superfluid models of NS cores involves using **multifluid hydrodynamics** $\longrightarrow$ **additional microscopic inputs**.

Entrainment effects (or *Andreev-Bashkin effects*)

Entrainment effects

- **Non-dissipative coupling** firstly discovered in context of ^{3}He-^{4}He (superfluid) mixtures.
- Due to **interactions**, the **superfluid flow** of the **first component entrains** the **superfluid flow** of the **second component and vice-versa**.

Nuclear physics case : respective **(mass) currents** of neutrons and protons ($\boldsymbol{\rho_n}$,$\boldsymbol{\rho_p}$) are linked to their respective **superfluid velocities** ($\boldsymbol{V_n}$, $\boldsymbol{V_p}$) and the normal fluid $\boldsymbol{V_{ex}}$ by :

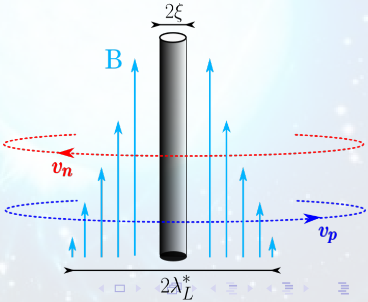
$$\boldsymbol{\rho_n} = mn_n \boldsymbol{V_{ex}} + \rho_{nn}\left(\boldsymbol{V_n} - \boldsymbol{V_{ex}}\right) + \rho_{np}\left(\boldsymbol{V_p} - \boldsymbol{V_{ex}}\right) \neq mn_n \boldsymbol{V_n}$$

$$\boldsymbol{\rho_p} = mn_p \boldsymbol{V_{ex}} + \rho_{pp}\left(\boldsymbol{V_p} - \boldsymbol{V_{ex}}\right) + \rho_{pn}\left(\boldsymbol{V_n} - \boldsymbol{V_{ex}}\right) \neq mn_p \boldsymbol{V_p}$$

$\rho_{qq'}$ = **Entrainment coefficients** = Intensity of the $q - q'$ coupling $(q, q' = n, p)$.

Entrainment effects leave their **imprints** in the **global dynamics** of neutron stars.

- **Vortex** in the neutron superfluid **entraining protons** $\longrightarrow$ **Induced magnetic field** $\boldsymbol{B}$.
- Electrons and muons scattering off this magnetic field $\longrightarrow$ e,μ-n **coupling**.



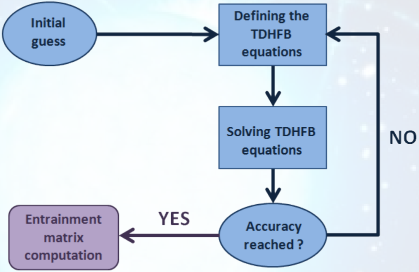
Energy-density functional theory (Nuclear EDFT)

How to **compute microscopically** the **entrainment matrix** ? ⟶ Use the **tools of condensed matter theory**.

Nuclear energy-density functional theory

The total **energy E** of a system can be written as a **functional** of the **density matrix** $n(\mathbf{r}\sigma, \mathbf{r}'\sigma', t)$.

- **Minimizing energy** E (for fixed temperature T and nucleon number) ⟶ **Time-dependent Hartree-Fock Bogoliubov (TDHFB) equations.**
- TDHFB equations are **self-consistent and highly non-linear !**
- **Continuity equation** can be derived from TDHFB equations **giving** the **mass current** $\boldsymbol{\rho_q}$ ⟶ Expression of **entrainment matrix** $\rho_{qq'}$!

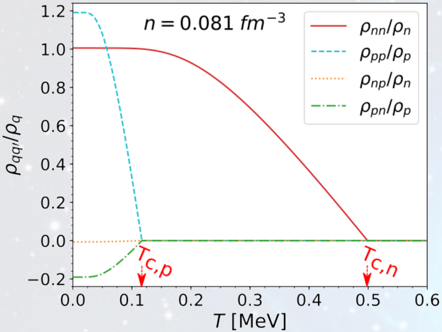


BUT **one needs to chose the energy-density functionnal E describing the neutron-proton system !**

Use of **phenomenological functionals** : functionals **adjusted** to **nuclear experiments** (nuclei masses, nuclear matter compressibility, heavy-ion collisions, etc.), **N-body computations** (e.g : neutron-matter EoS, symmetry energies) and **observations** (maximal mass of NS).

Entrainment matrix at crust-core transition and conclusion

Solving TDHFB at **crust-core transition $\mathbf{n_{cc} = 0.081\ fm^{-3}}$** for **low superfluid flows compared to the velocities required to break superfluidity** (for simplicity),



- Note : $\rho_q = mn_q$ (q=n,p) is the mass density. $\mathbf{T_{c,n} = 0.5\ MeV}$ and $\mathbf{T_{c,p} = 0.12\ MeV}$ are the neutron/proton critical temperatures.
- $\mathbf{T \ll T_{c,n}, T_{c,p}}$: Entrainment matrix **independent on temperature**.
- For $\mathbf{T_{c,p} \leq T < T_{c,n}}$: Proton superfluid disappearance ($\longrightarrow \rho_{np} = \rho_{pn}$ **and** ρ_{pp} **vanish**).
- For $\mathbf{T \geq T_{c,n}}$: Neutron superfluid disappearance ($\longrightarrow \rho_{nn}$ **vanishes**).

Conclusions and perspectives

- **Generalization** of **previous studies** using T = 0K or low temperatures approximations.
- Perpectives and applications : **magnetic flux of neutron vortices**/pulsar dynamics, **NS oscillations**, **improvement of nuclear EDF** implementing currents and improve the study of nuclear matter at high densities.
- **Further information :** `https://doi.org/10.1103/PhysRevC.100.065801` & `https://arxiv.org/abs/2006.15317`